MOTION OF A MAGNETIZABLE LIQUID IN A ROTATING MAGNETIC FIELD

V. M. Suyazov

Moskowitz and Rosensweig [1] describe the drag of a magnetic liquid – a colloidal suspension of ferromagnetic single-domain particles in a liquid carrier – by a rotating magnetic field. Various hydrodynamic models have been proposed [2, 3] to describe the macroscopic behavior of magnetic suspensions. In the model constructed in [2] it was assumed that the intensity of magnetization is always directed along the field so that the body torque is zero. Therefore, this model cannot account for the phenomenon under consideration. We make a number of simplifying assumptions to discuss the steady laminar flow of an incompressible viscous magnetizable liquid with internal rotation of particles moving in an infinitely long cylindrical container in a rotating magnetic field. The physical mechanism setting the liquid in motion is discussed. The importance of unsymmetric stresses and the phenomenon of relaxation of magnetization are emphasized. The solution obtained below is also a solution of the problem of the rotation of a polarizable liquid in a rotating electric field* according to the model in [3].

1. Neglecting electrical conductivity, polarizability, and cross effects, the equations [4] for an isothermal incompressible liquid with constant physical properties consist of the momentum equation (1.1),

*While this paper was in press an article by V. M. Zaitsev and M. I. Shliomis [4] on a similar problem was published. The main conclusions of the present paper agree with those of [4]. The papers differ in their approach to the description of the drag of a ferromagnetic suspension by a rotating field.

In [4] the problem is solved by using the equations of motion of an incompressible liquid having an intrinsic angular momentum [5], taking account of the macroscopic external torque per unit volume. In general, this system of equations is not closed. For closure it is necessary first to know the magnitude of the macroscopic external torque, and in [4] it is proposed to calculate this torque by averaging the microscopic moments acting on each individual particle; each specific case requires special considerations [4]. Secondly, it is necessary to use another equation, e.g., from the theory of paramagnetic relaxation, describing the change of the intensity of magnetization with time.

Thus, the method used in [4] to solve the problem of the rotation of a ferromagnetic suspension is not based on a pure hydrodynamic model which could be used to describe the mechanical behavior of such liquids. Therefore, it is of interest to describe the phenomenon in question within the framework of a model of a magnetizable liquid constructed by using the methods of the mechanics of continuous media [6]. The present paper is concerned with finding just such a solution. The solution found shows the possibility of using the equations in [3] to account for the macroscopic behavior of ferromagnetic suspensions. The expressions for the electromagnetic body torque obtained in [3] and the equation describing the change of the intensity of magnetization with time are rather general and are applicable to a broad class of physicomechanical phenomena.

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the equation for the internal rotation of the particles (1.2), the equation for the magnetization (1.3), and the equations of the electrodynamics of a moving magnetizable medium (1.4):

$$\rho \mathbf{v}^{\cdot} = -\nabla p + 2\alpha_2 \nabla\cdot(\nabla \mathbf{v})^d + \alpha_3 \nabla\times(2\boldsymbol{\omega} - \nabla\times\mathbf{v}) + \mu_0 \mathbf{m}\cdot\nabla\mathbf{h} - \mu_0 \mathbf{m}^{\cdot}\times\varepsilon_0\mathbf{e} - \mathbf{v}\times(\mu_0\mathbf{m}\cdot\nabla)\,\varepsilon_0\mathbf{e} + \rho\mathbf{f} \tag{1.1}$$

$$p = -(\partial\varphi/\partial\rho^{-1}) + K^{-1}\mu_0 m^2, \qquad \nabla\cdot\mathbf{v} = 0$$

$$\rho J \boldsymbol{\omega}^{\cdot} = \gamma_1 \nabla\nabla\cdot\boldsymbol{\omega} + 2\gamma_2\nabla\cdot(\nabla\boldsymbol{\omega})^d + 2\gamma_3\nabla\cdot(\nabla\boldsymbol{\omega})^a + 2\alpha_3(\nabla\times\mathbf{v} - 2\boldsymbol{\omega}) + \mu_0\mathbf{m}\times\boldsymbol{\eta} \tag{1.2}$$

$$\boldsymbol{\eta} - K^{-1}\mathbf{m} = \mu_0 h_1(\mathbf{m}^{\cdot} - \boldsymbol{\omega}\times\mathbf{m}), \qquad \boldsymbol{\eta} = \mathbf{h} - \mathbf{v}\times\varepsilon_0\mathbf{e} \tag{1.3}$$

$$\nabla\times\mathbf{e} + \partial\mu_0\mathbf{h}/\partial t = -\partial\mu_0\mathbf{m}^{\cdot}/\partial t - \nabla\times(\mu_0\mathbf{m}\times\mathbf{v}), \qquad \nabla\cdot\varepsilon_0\mathbf{e} = 0 \tag{1.4}$$

$$\nabla\times\mathbf{h} - \partial\varepsilon_0\mathbf{e}/\partial t = 0, \qquad \nabla\cdot\mu_0\mathbf{h} = -\nabla\cdot\mu_0\mathbf{m}$$

Here ρ is the mass density, **v** is the velocity, $\boldsymbol{\omega}$ is the angular velocities of the particles, p is the hydrostatic pressure, J is the average value of the moment of inertia of a unis mass, **m** is the intensity of magnetization, **e** and **h** are the electric and magnetic field intensities, ε_0 and μ_0 are the permittivity and permeability of vacuum, ∇ is the del operator, φ is the free energy, **f** is the body force, α_2 is a viscosity coefficient, α_3 is the coefficient of rotational viscosity characterizing the contribution of the unsymmetrical stresses to the change in momentum, γ_1, γ_2, and γ_3 are shear-stress viscosity coefficients, $K > 0$ is the magnetic susceptibility of a paramagnet, and $\tau = \mu_0 h_1 K$ is the relaxation time of the magnetization. The superscript d denotes the deviatoric parts of symmetric dyads, and the superscript a denotes anitsymmetric dyads. A dot above and to the right of a quantity denotes its total time derivative.

Equations (1.1)-(1.4) form a closed system of equations describing the motion of an isotropic homogeneous magnetizable liquid and take account of the internal rotation of the particles and the phenomenon of magnetic relaxation.

2. We assume that a magnetizable liquid with internal rotation of particles is contained in an infinitely long cylinder of radius R. A magnetic field of constant intensity h_0 is applied perpendicular to the axis of the cylinder and rotated with a constant angular velocity ω_0.

We solve the problem in cylindrical coordinates r, θ, z rotating with an angular velocity equal to that of the magnetic field.

In solving the problem we neglect the change in magnetic and electric fields connected with the motion of the magnetizable liquid. Therefore, the components of the magnetic field both for the region occupied by the liquid and the region free of liquid have the form

$$h_r = h_0\cos\theta, \quad h_\theta = -h_0\sin\theta, \quad h_z = 0 \tag{2.1}$$

The magnetic field (2.1) tends to move ferromagnetic particles in the direction of rotation. Because of viscous resistance in the liquid surrounding the particles their rotational motion produces a phase difference δ between the direction of the field and that of the intensity of magnetization produced by the field. Therefore, we have

$$m_r = m_0\cos(\theta + \delta), \quad m_\theta = -m_0\sin(\theta + \delta), \quad m_z = 0 \tag{2.2}$$

Here m_0 is the intensity of magnetization.

The boundary conditions are determined by finite values of motion velocity and of velocity of internal rotation around the axis of the cylinder, the adhesion of the liquid to the cylinder wall, and the absence of internal rotation at the surface of the cylinder

$$\mathrm{v}(0) \neq \infty, \quad \omega(0) \neq \infty, \quad \mathrm{v}(R) = -R\omega_0, \quad \omega(R) = -\omega_0 \tag{2.3}$$

Stationary flow under these conditions is described by a solution of Eqs. (1.1)-(1.3) of the form

$$\begin{gathered} v_\theta = v(r), \quad \omega_z = \omega(r), \quad m_0 = m_0(r), \quad \delta = \delta(r) \\ p = p(r), \quad v_r = v_z = \omega_r = \omega_\theta = 0 \end{gathered} \tag{2.4}$$

From (1.1)-(1.3) we find the following relations for determining v, w, m_r, m_θ, and p:

$$\begin{gathered} \alpha^*\frac{1}{r}\frac{d(rv)}{dr} - 2\alpha_3\omega = A, \quad \frac{dp}{dr} = \frac{\rho v^2}{r} + 2\rho v\omega_0 + \rho\omega_0^2 r \\ \gamma^*\frac{1}{r}\frac{d}{dr}\left(r\frac{d\omega}{dr}\right) + 2\alpha_3\left(\frac{1}{r}\frac{d(rv)}{dr} - 2\omega\right) + M = 0 \end{gathered} \tag{2.5}$$

$$M = \mu_0 (m_r h_\theta - m_\theta h_r), \quad \alpha^* = \alpha_2 + \alpha_3, \quad \gamma^* = \gamma_2 + \gamma_3$$
$$Kh_r = m_r + \tau\left[\frac{v}{r}\left(\frac{\partial m_r}{\partial \theta} - m_\theta\right) + m_\theta \omega\right] \tag{2.6}$$
$$Kh_\theta = m_\theta + \tau\left[\frac{v}{r}\left(\frac{\partial m_\theta}{\partial \theta} + m_r\right) - m_r \omega\right]$$

Here A is an arbitrary integration constant.

By using (2.1) and (2.2) we find from (2.6)

$$\operatorname{tg} \delta = -\tau\omega, \qquad m_0 = Kh_0 \,/\, (1 + \operatorname{tg}^2 \delta) \cos \delta \tag{2.7}$$

Henceforth we limit ourselves to a consideration of liquids with small viscosity coefficients and to such an angular velocity of the rotating field that the angle of lag is appreciably smaller than unity ($\delta \ll 1$). In this case, (2.7) and the torque equation (2.5) take the form

$$\delta = -\tau\omega, \qquad m_0 = Kh_0, \qquad M = \mu_0 K h_0^2 \delta \tag{2.8}$$

By using (2.8) Eqs. (2.5) can be put in the form

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\omega}{dr}\right) - k^2\omega + bA = 0, \qquad k^2 = \frac{4\alpha_2\alpha_3 + \tau^*\alpha^*}{\alpha^*\gamma^*} \tag{2.9}$$
$$\frac{1}{r}\frac{d(rv)}{dr} = \frac{A}{\alpha^*} + b\gamma^*\omega, \qquad b = \frac{2\alpha_3}{\alpha^*\gamma^*}, \qquad \tau^* = \tau K h_0^2 \mu_0$$

In Eqs. (2.9) the quantity k^2 is positive. This follows from the form of the thermodynamic constraints on the phenomenological coefficients [3].

Equations (2.9) and the boundary conditions (2.3) have the solution

$$v = -r\omega_0 + \frac{Bb\gamma^*}{k}\left(I_1(kr) - \frac{r}{R} I_1(kR)\right) \tag{2.10}$$
$$\omega = -\frac{2b\alpha^*\omega_0}{k^2 + b^2\gamma^*\alpha^*} + B\left(I_0(kr) - \frac{2\alpha^* b^2\gamma^*}{kR\,(k^2 + b^2\gamma^*\alpha^*)} I_1(kR)\right)$$
$$B = \omega_0\left(\frac{2b\alpha^*}{k^2 + b^2\gamma^*\alpha^*} - 1\right)\left(I_0(kR) - \frac{2\alpha^* b^2\gamma^*}{kR\,(k^2 + b^2\gamma^*\alpha^*)} I_1(kR)\right)^{-1} \tag{2.11}$$

Here I_0 and I_1 are modified Bessel functions of the first kind.

Thus it is possible to account qualitatively for the rotation of a ferromagnetic liquid in a rotating magnetic field [1] within the framework of the nonsymmetric model of an electromagnetic liquid [3]. This model suffices because it takes account of the dissipation of electromagnetic-field energy in relaxation processes related to the phenomena of polarization and magnetizability.

It is clear from (2.6) that the presence of terms containing the magnetization relaxation time leads to a difference in the directions of the magnetic field and the intensity of magnetization. This gives rise to a body torque (2.8) which induces an internal rotation of the particles in the liquid (2.5).

As a consequence of the viscosity of the liquid the rotation of particles by the field causes a drag of the liquid surrounding the particles. Thus local vortices are induced in the liquid. The presence of "friction" ($\alpha_3 \neq 0$) between the internal rotation field $\boldsymbol{\omega}$ and the external rotation field $\nabla \times \mathbf{v}$ arising from the difference in inertial properties of the liquid and particles produces nonsymmetric stresses (2.5) which set the liquid into macroscopic motion — into rotation (2.10).

To explain the role of shear stresses in the appearance of macroscopic motion of the liquid we set $\gamma^* = 0$ in (2.5). Then Eqs. (2.5) and the first and third boundary conditions of (2.3) have the solution

$$v \equiv 0, \qquad \omega = \tau^* \, (4\varkappa_3 + \tau^*)^{-1} \, \omega_0 \tag{2.12}$$

in the stationary reference frame.

It is clear from (2.12) that there is no macroscopic motion in the liquid and that the internal rotation of the particles forms a uniform field. Thus the macroscopic motion of the liquid is essentially connected with the nonuniformity of the internal rotation field of the particles, i.e., with the presence of shear stresses. It follows from (1.1) that only under this condition do asymmetric stresses appear in the liquid which contribute to the change in momentum and set the liquid in motion.

We note that if we replace the magnetic susceptibility and the magnetic-field intensity in Eqs. (2.10) and (2.11) by the dielectric susceptibility and the electric-field intensity, respectively, we obtain a solution of the problem of the rotation of a polarizable liquid in a rotating electric field according to the model in [3].

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